

Roll No.

Total Pages : 04

BT-3/D-20
43135
MATHEMATICS-III
BS-205A

Time : Three Hours]

[Maximum Marks : 75

Note : All questions in Part A and Part B are compulsory.Attempt any *four* questions from Part C, selecting *one* question from each Unit.**Part A**

1. (a) Determine the following series converges or

diverges $\sum_{n=2}^{\infty} \frac{1}{n^5 - n^2 - 1}$.

(b) Solve $3y' + xy = xy^{-2}$.

- (c) Find the solution of :

$$(D^2 + 4D + 4)y = 5 \cos x.$$

- (d) Evaluate the integral :

$$\int_0^1 \int_0^x \int_0^{1+2x+3y} f(x, y, z) dx dy dz, \text{ where } f(x, y, z) = 5.$$

- (e) Evaluate Curl of
- $e^{xyz}(i + j + k)$
- at the point (1, 2, 3).

5×3=15

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Part B

2. Determine whether the series converge :

(a) $\sum_{n=1}^{\infty} \frac{n}{e^n}$.

(b) $\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{5^n}$.

3. Solve :

$$\left(y + \sqrt{x^2 + y^2} \right) dx - x dy = 0, y(1) = 0.$$

4. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dz \, dy \, dx$.

5. Calculate
- $\nabla^2 f$
- when
- $f = 3x^2z - y^2z^3 + 4x^3y + 2x - 3y - 5$
- at the point (1, 1, 0).

Part C**Unit I**

6. (a) Define Cauchy first root test for sequence. Also check the convergence of
- $\langle a_n \rangle$
- , where

$$a_n = \left(\frac{n^3 + n}{n+5} \right)^{\frac{1}{n}}.$$

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- (b) Series
- $\sum_{n=1}^{\infty} \frac{1}{n!}$
- converges or diverges ? Justify.
- 5**

7. Expand
- $f(x) = x \sin x$
- as a Fourier series in
- $(0, 2\pi)$
- .
- 10**

Unit II

8. (a) Find the general solution and singular solution of
- $y = \sin(y - xp)$
- .
- 5**

- (b) Solve
- $y(2x^2 - xy + 1)dx + (x - y)dy = 0$
- using exact differential equation.
- 5**

9. (a) Solve :

$$(D^2 - 4D - 5)y = e^{2x} + 3 \cos(4x + 3)$$

- (b) Solve the following differential equation using method of variation of parameter

$$(D^2 - 2D)y = e^x \sin x. \quad \mathbf{5}$$

Unit III

10. Evaluate
- $\iint \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}$
- over the positive quadrant of the circle
- $x^2 + y^2 = 1$
- .
- 10**

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11. Find the volume of the portion of the sphere
- $x^2 + y^2 + z^2 = a^2$
- lying inside the cylinder
- $x^2 + y^2 = ay$
- .
- 10**

Unit IV

12. Show that
- $\nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$
- for any vector function
- $\bar{A}$
- .
- 10**

13. Verify the Stokes theorem for
- $\bar{A} = y^2i + xyj + xzk$
- where S is the hemisphere
- $x^2 + y^2 + z^2 = a^2, z \geq 0$
- .
- 10**